Design of an Extrusion Die With a Variable Choker Bar

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SYNOPSIS

A novel approach is proposed to adjust flow nonuniformities caused by production variations from an extrusion die. A mathematical model that is based on the lubrication approximation was developed, and the effect of four types of production variations was examined separately. To each production variation, a specially designed choker bar was constructed and inserted into the extrusion die to correct flow nonuniformities. © 1994 John Wiley & Sons, Inc.

INTRODUCTION

Extrusion dies are employed to deliver thin liquid layers for the production of wide polymer and metal films or sheets and also appear in many coating operations. Two types of extrusion dies, T-dies and coat-hanger dies, are frequently used. A T-die is easy to construct, but a coat-hanger die has more advantages in terms of acceptable flow uniformity and narrow residence time distributions. Many authors have proposed mathematical analyses to predict the fluid motion inside extrusion dies. The analyses can be categorized into two types. The first type is called the lubrication approximation, which is based on the one-dimensional or two-dimensional macroscopic mass and momentum balances.[1-21] The second type is based on the three-dimensional finite element simulation to determine the velocity and pressure fields inside an extrusion die.[22-25]

Most of the previous studies were concerned with the design of an extrusion die that delivers a wide and thin sheet with acceptable lateral uniformity for a particular polymeric liquid. However, because of high precision requirements, extrusion dies are very expensive to construct; and it is not economical to build a die that is only suitable for a particular polymeric liquid. In practice, an extrusion die should be flexible to meet many production variations. Liu et al.[7] found that a linearly tapered coat-hanger die is very sensitive to different production variations, but they did not propose any remedies to correct flow nonuniformities caused by production variations.

In this article we discuss a novel and simple approach to correct flow nonuniformities for each production variation. Instead of rebuilding dies to meet production variations, we constructed a specially designed choker bar for each production variation; the flow nonuniformities can be properly adjusted by inserting this choker bar into the extrusion die. Traditionally a choker bar has a rectangular cross section and is used manually through a trial and error manner to correct flow nonuniformities for an extrusion die. In this article we present design formulas that can be used to construct tapered choker bars so that trial and error approaches can be eliminated and to produce an extrusion die that has more flexibility to handle different production requirements.

We shall consider the following production variations:

1. The optimal manifold shape based on the lubrication approximation to deliver uniform liquid sheets is tapered and the cross-sectional area of the manifold at the ends of the die is very small.[7] The ideal manifold is not only difficult to construct, but may also create many practical production problems such as cleaning and flushing. Therefore the manifold has to be enlarged and the deterioration of

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flow uniformity because of the enlargement should be properly compensated.

2. An extrusion die should be suitable for a wide range of flow rates. As flow rates increase, the inertial effect can be significant, particularly for metal extrusion and some coating operations, and should be taken into consideration.
3. An extrusion die should be flexible for polymeric liquids with different rheological behavior because when the product formulations are updated, the rheologies of the associated polymeric liquids are also varied.
4. An extrusion die should deliver uniform liquid films or sheets with different widths to meet production specifications.

A mathematical model that is based on the lubrication approximation was developed to analyze the effect of these four types of variations, and

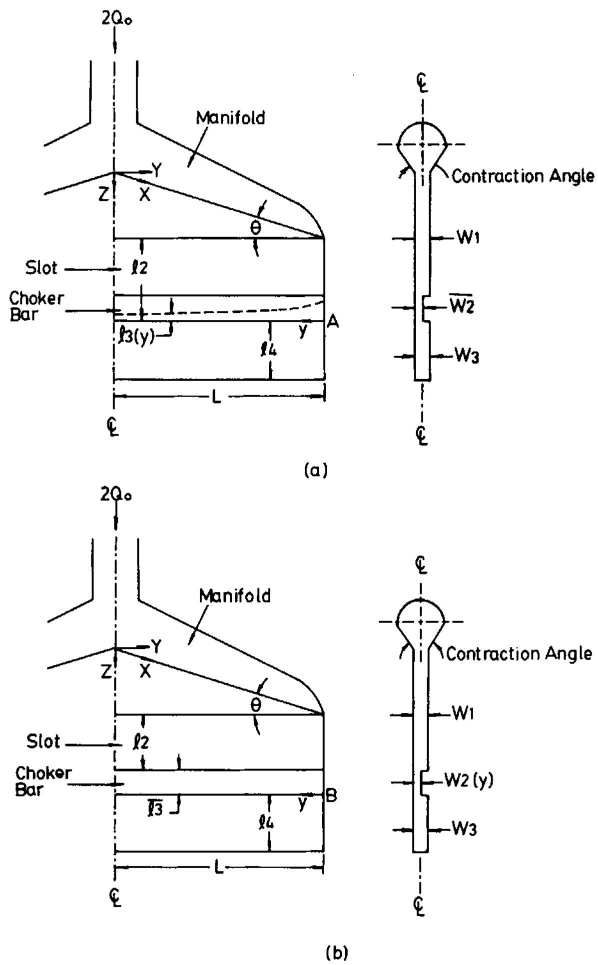

Figure 1 A linearly tapered coat-hanger die with an adjustable choker bar. (a) Die I; (b) Die II.

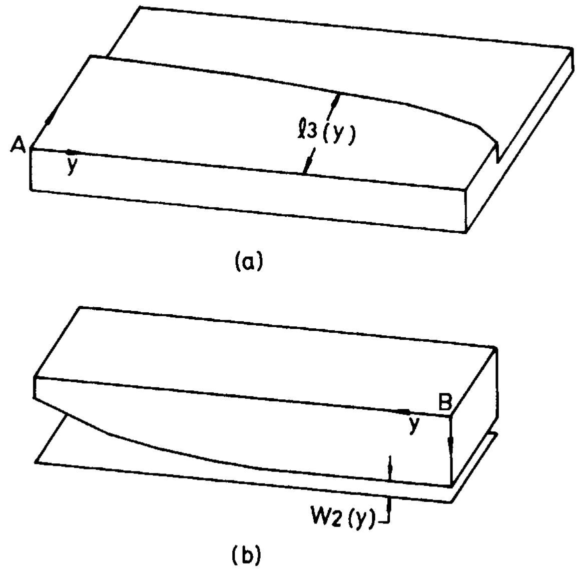

Figure 2 Two different shapes of choker bar: (a) used in Die I; (b) used in Die II.

choker bars with specially tapered shapes were constructed to correct flow nonuniformities due to these variations.

EXPERIMENTAL

Mathematical Formulation

The extrusion die we consider is a center-fed linearly tapered coat-hanger die as shown in Figure 1. Because of symmetry, we shall only examine the right half of the die. A liquid enters the interior of the die through the inlet tube; after filling the manifold, the liquid moves forward through the slot section and forms a wide and thin liquid layer. A choker bar may be positioned in the middle of the slot section to adjust flow uniformity by controlling the slot gap. Traditionally, the choker bar has a regular rectangular cross section and is adjusted manually through a trial and error procedure to correct flow nonuniformities. Here we used a choker bar with a specially designed geometry to minimize the effect of four production variations mentioned earlier. The choker bar we propose may have two different geometries (Fig. 2); the first type in Figure 2(a) or Die I, is a two-step choker bar, the shape function $l(y)$ will be derived later. Note that the positions of point A in Figures 1 and 2 are different because we turned the choker bar upside down to better illustrate the shape function $l(y)$. The second type, or Die II in Figure 2(b), has a tapered section, and the tapered function $w(y)$ will also be derived later. The major objective of this paper is to select $l(y)$ and $w(y)$ properly to eliminate flow nonuniformities caused by production variations.

We shall consider the isothermal and laminar fluid motion inside the die; the entrance and the end effects are neglected. The liquid under consideration is assumed to obey the power-law model and the viscosity can be represented as[26]:

$$\eta = k\gamma^{n-1}. \tag{1}$$

If $n = 1$, the fluid is Newtonian; if $n < 1$, the fluid will exhibit the shear-thinning behavior quite common for most polymeric liquids.

We apply the lubrication approach to determine the flow distribution inside the die. Lee and Liu[10] derived the pressure drop/flow rate equation in the manifold as follows:

$$\frac{dp}{dY} = \frac{-2\rho\beta}{\bar{h}^4}\bar{Q}\frac{d\bar{Q}}{dY} + \frac{4\rho\beta}{\bar{h}^5}\bar{Q}^2\frac{d\bar{h}}{dY} - \frac{k\bar{Q}^n}{\lambda^n\bar{h}^{3n+1}\cos\theta} \tag{2}$$

where the first two terms in the right-hand side of eq. (2) represent the inertial effect, and the third terms represent the effect of viscous force. β is a kinematic factor and λ is a shape factor, both depend on the shape of the manifold and the power-law index n. Numerical values of β and λ for various manifold shapes can be found elsewhere.[10] The gravitational effect is neglected here. The volumetric flow rate per unit die width in the slot section can be represented as follows[7]:

$$\bar{q} = \frac{w^2}{E}\left(\frac{w}{k}\right)^{1/n}\left(\frac{-dp}{dZ}\right)^{1/n} \tag{3}$$

$$E = 2^{1+(1/n)}\left(2 + \frac{1}{n}\right) \tag{4}$$

where w represents w_1, $\bar{w}_2$, or w_3 in Figure 1. The slot gap may be variable in the whole slot section as indicated in Figure 1; the pressure drop in the slot section can be estimated by evaluating the pressure drop in each part with the same slot gap through the lubrication approximation and then adding the total pressure drop together. This procedure was adopted from two previous studies.[2,27] For Die I, the pressure drop in the slot section is

$$-\frac{dp}{dZ} = \frac{p}{F_1} \tag{5}$$

where

$$F_1 = (L - Y)g + l_2 - \bar{l}_3 + f_2\bar{l}_3 + f_3 l_4 \tag{6}$$

and we take the reference pressure to be zero.

For Die II, the pressure drop in the slot section is

$$-\frac{dp}{dZ} = \frac{p}{F_2} \tag{7}$$

where

$$F_2 = (L - Y)g + l_2 + f_2\bar{l}_3 + f_3 l_4 \tag{8}$$

and

$$g = \tan\theta, \quad f_2 = \left(\frac{w_1}{\bar{w}_2}\right)^{2n+1}, \quad f_3 = \left(\frac{w_1}{w_3}\right)^{2n+1}. \tag{9}$$

Because the decrease of the volumetric flow rate in the manifold is equal to the amount that leaks into the slot section, we have the following material balance

$$\frac{d\bar{Q}}{dY} = -\bar{q}. \tag{10}$$

We now define the dimensionless variables as follows:

$$q = \bar{q}/(Q_0/L), \quad Q = \bar{Q}/Q_0, \quad y = 1 - Y/L, \quad h = \bar{h}/h_0 \quad l_3 = \bar{l}_3/l_{30}, \quad w_2 = \bar{w}_2/w_{20}. \tag{11}$$

Here the subscript 0 denotes values at the center line $y = 0$. Differentiating eq. (10) with respect to y and substituting eqs. (5) or (6) into the resulting equations, we obtain equations that govern the volumetric flow rates in the manifolds in Die I and Die II. For Die I, we obtain

$$\frac{d^2Q}{dy^2} + \frac{\left[1 + (f_2 - 1)\dfrac{l_{30}}{gL}\dfrac{dl_3}{dy}\right]\dfrac{dQ}{dy}}{nF_3} + \frac{\mathrm{ReNm}Q\left(\dfrac{dQ}{dy}\right)^{2-n}}{nF_3h^4} - \frac{\mathrm{ReNn}Q^2\left(\dfrac{dQ}{dy}\right)^{1-n}}{nF_3h^4} - \frac{\mathrm{Nv}Q^n\left(\dfrac{dQ}{dy}\right)^{1-n}}{nF_3h^{3n+1}} = 0. \tag{12}$$

For Die II, we obtain

$$\frac{d^2Q}{dy^2}+\frac{\left(1+\frac{\bar{l}_3}{gL}\frac{df_2}{dy}\right)\frac{dQ}{dy}}{nF_4}+\frac{\mathrm{ReNm}Q\left(\frac{dQ}{dy}\right)^{2-n}}{nF_4h^4}-\frac{\mathrm{ReNn}Q^2\left(\frac{dQ}{dy}\right)^{1-n}}{nF_4h^4}-\frac{\mathrm{Nv}Q^n\left(\frac{dQ}{dy}\right)^{1-n}}{nF_4h^{3n+1}}=0. \quad (13)$$

Note here that the Reynolds number Re is defined as

$$\mathrm{Re}\equiv\frac{\rho(Q_0/Lw_1)^{2-n}w_1^n}{k} \quad (14)$$

and

$$\mathrm{Nm}\equiv\frac{2\beta w_1^3L}{E^nh_0^4g},\quad \mathrm{Nn}\equiv\frac{4\beta w_1^3L(dh/dy)}{E^nh_0^4g}$$

$$\mathrm{Nv}\equiv\frac{w_1^{2n+1}L^n}{E^n\lambda^nh_0^{3n+1}\sin\theta}$$

$$F_3=\left(y+\frac{l_2-l_{30}l_3+f_2l_{30}l_3+f_3l_4}{gL}\right)$$

$$F_4=\left(y+\frac{l_2+f_2\bar{l}_3+f_3l_4}{gL}\right). \quad (15)$$

The dimensionless boundary conditions for the flow equations are

$$y=0,\quad Q=0$$
$$y=1,\quad Q=1. \quad (16)$$

The flow equations can be discretized by standard finite-difference schemes and then solved iteratively by the Newton–Raphson method. The detailed numerical procedure is similar to the work of Liu et al.[7] and will not be reported here. Once Q is determined, q can be computed using eq. (10). Note q represents the lateral flow uniformity. For a uniform liquid layer, q is equal to unity and then $Q = y$.

If the choker bars in Die I and Die II are removed, the slot sections in both dies will have a constant gap w, then eqs. (12) and (13) reduce to the flow equation developed by Liu et al.[7] For a given liquid, if we fix all the geometric variables except h and substitute the uniform flow condition $Q = y$ into eq. (13), we obtain

$$h=y^{n/3n+1},$$

$$h_0=\left(\frac{Lw_1^{2+1/n}}{\lambda E}\right)^{[n/(3n+1)]}(\csc\theta)^{[1/(3n+1)]} \quad (17)$$

which implies if the manifold is tapered following eq. (17), the die will deliver a uniform liquid film or sheet.

We now discuss how to design choker bars properly so that the flow nonuniformities caused by the four production variations can be corrected.

Manifold Enlargement

It can be observed from eq. (17) that h becomes very small at the ends of the die; this creates a maintenance problem if the interior of the die has to be cleaned. Therefore in practice the manifold has to be enlarged at the ends of the die. A simple way to enlarge the manifold is to select h as follows:

$$h=(y+y_0)^{[n/(3n+1)]}. \quad (18)$$

Here y_0 is an adjustable constant. Therefore as y approaches 0, h is not small and the maintenance problem can be solved. If the manifold is constructed based on eq. (18), the flow uniformity no longer exists. To regain this uniformity, we need to taper the choker bar properly. We fix all the geometric parameters of Die I except l_3 and assign $Q = y$, which implies the flow distribution will be uniform. By substituting eq. (18) into the flow equation (12) for Die I, we obtain

$$(f_2-1)\frac{l_{30}}{gL}\frac{dl_3}{dy}=\mathrm{ReNn}\frac{y^2}{(y+y_0)^{4n/3n+1}}+\left(\frac{y}{y+y_0}\right)^n-\mathrm{ReNm}\frac{y}{(y+y_0)^{4n/3n+1}}-1 \quad (19)$$

with boundary condition $y = 0$ and $l_3 = 1$. Equation (19) can be numerically integrated to generate the taper function $l_3(y)$. Once a choker bar is constructed following the design equation $l_3(y)$, this bar can be inserted into the die to correct the flow nonuniformities caused by manifold enlargement. Similarly, we substitute eq. (18) into eq. (13) and obtain

$$(2n+1)\frac{\bar{l}_3}{gL}\left(\frac{w_1}{w_{20}}\right)^{2n+1}\frac{dw_2}{dy}=w_2^{2n+2}\left[1-\left(\frac{y}{y+y_0}\right)^n+\mathrm{ReNm}\frac{y}{(y+y_0)^{4n/3n+1}}-\mathrm{ReNn}\frac{y^2}{(y+y_0)^{4n/3n+1}}\right] \quad (20)$$

with boundary condition $y = 0$ and $w_2 = 1$. Therefore once $w_2(y)$ is obtained, a choker bar can be built to eliminate flow nonuniformities.

The enlarged manifold should be used in considering the following production variations.

Inertial Effect

The viscosity of polymeric liquids is usually very high, so the inertial force can be neglected. However, for metal extrusion and coating operations, the fluid viscosity can be low and the inertial force cannot be neglected. Leonard[8] and Lee and Liu[10] used the one-dimensional lubrication approach to analyze the effect of inertial force and their results showed that the inertial force will deteriorate the flow uniformity. To regain perfect flow distribution in Die I, we need to substitute eq. (18) into eq. (12) and select $Q = y$. The taper function $l_3(y)$ is the same as eq. (19). Once a choker bar is constructed with the taper function $l_3(y)$ for a given Re, the nonuniformities caused by the inertial force can be eliminated. Similarly, the taper function $w_2(y)$ is the same as (20) and the choker bar can be built accordingly.

Correction for Solutions With Different Rheological Properties

An extrusion die is usually designed for a particular polymeric liquid. Liu et al.[7] found that the flow uniformity is quite sensitive to the rheological properties of the delivered polymeric liquid. If the rheological properties vary, the flow uniformity will deteriorate rapidly. Because the die is very expensive, it is desirable to use a die for solutions with different rheological properties. If a die is designed based on a polymeric liquid with the power-law index m, then the manifold taper function h is

$$h = (y + y_0)^{[m/(3m+1)]}$$

$$h_0 = \left[\frac{Lw_1^{2+1/m}}{\lambda(m)E(m)}\right]^{[m/(3m+1)]} (\csc\theta)^{[1/(3m+1)]}. \quad (21)$$

Now the die is used to deliver a solution with a different power-law index n and the flow uniformity has to be restored. To regain flow uniformity in Die I, we need to substitute eq. (21) into eq. (12) while $Q = y$, then we obtain

$$(f_2 - 1)\frac{l_{30}}{gL}\frac{dl_3}{dy} = \mathrm{ReNn}\frac{y^2}{(y+y_0)^{4m/3m+1}} + \frac{y^n D}{(y+y_0)^{\delta}} - \mathrm{ReNm}\frac{y}{(y+y_0)^{4m/3m+1}} - 1 \quad (22)$$

with boundary condition $y = 0$ and $l_3 = 1$ where

$$D = \left(\frac{\csc^3\theta w_1}{L}\right)^{[(m-n)/(3m+1)]} \cdot \frac{[\lambda(m)E(m)]^{\delta}}{[\lambda(n)E(n)]^{n}}$$

$$\delta = m\frac{3n+1}{3m+1}. \quad (23)$$

The taper function $l_3(y)$ can be obtained by numerically integrating eq. (22). A choker bar with the taper function $l_3(y)$ can be built to correct flow nonuniformities because of viscosity variations. Similarly, substituting eq. (21) into eq. (13) with $Q = y$, we obtain for Die II

$$(2n+1)\frac{\bar{l}_3}{gL}\left(\frac{w_1}{w_{20}}\right)^{2n+1}\frac{dw_2}{dy} = w_2^{2n+2}\left[1 - \frac{y^n D}{(y+y_0)^{\delta}} + \mathrm{ReNm}\frac{y}{(y+y_0)^{4m/3m+1}} - \mathrm{ReNn}\frac{y^2}{(y+y_0)^{4m/3m+1}}\right] \quad (24)$$

with boundary condition $y = 0$ and $w_2 = 1$ and the taper function $w_2(y)$ for the adjustable choker bar can be found through a numerical integration.

Product Width Variations

We usually design a die that delivers the widest liquid layers as production needs. If it is necessary to produce films or sheets narrower than the designed die usually delivers, we can block the two ends of the die; however, the flow uniformity will be destroyed by doing so.

The flow equations (12) and (13) that describe the volumetric flow distributions in the manifold remain the same if the two ends of the die are blocked. However, the corresponding boundary conditions are different and are as follows:

$$y = \epsilon, \quad Q = 0$$
$$y = 1, \quad Q = 1. \quad (25)$$

Here ϵ is the dimensionless reduction of the production width. To regain perfect flow distribution, the volumetric flow rate Q should have the following form:

$$Q = \frac{y - \epsilon}{1 - \epsilon}. \quad (26)$$

Substituting eqs. (18) and (26) into eq. (13), we obtain for Die I

$$(f_2 - 1)\frac{l_{30}}{gL}\frac{dl_3}{dy} = \mathrm{ReNn}\frac{(y-\epsilon)^2(1/1-\epsilon)^{2-n}}{(y+y_0)^{4m/3m+1}} + \left(\frac{y-\epsilon}{y+y_0}\right)^n - \mathrm{ReNm}\frac{(y-\epsilon)(1/1-\epsilon)^{2-n}}{(y+y_0)^{4n/3n+1}} - 1 \tag{27}$$

with boundary condition $y = \epsilon$ and $l_3 = 1$. Therefore if we numerically integrate eq. (27) to generate the taper function $l_3(y)$ and then construct a choker bar using this $l_3(y)$ as the taper function, the flow distribution can be uniform again even if the two ends of the die are blocked.

Similarly, after substituting eqs. (18) and (26) into eq. (13), we obtain for Die II

$$(2n+1)\frac{\bar{l}_3}{gL}\left(\frac{w_1}{w_{20}}\right)^{2n+1}\frac{dw_2}{dy} = w_2^{2n+2}\left[1 - \left(\frac{y-\epsilon}{y+y_0}\right)^n + \mathrm{ReNm}\frac{(y-\epsilon)(1-\epsilon)^{2-n}}{(y+y_0)^{4n/3n+1}} - \mathrm{ReNn}\frac{(y-\epsilon)^2(1/1-\epsilon)^{2-n}}{(y+y_0)^{4n/3n+1}}\right] \tag{28}$$

with boundary condition $y = \epsilon$ and $w_2 = 1$. Once $w_2(y)$ is obtained, a choker bar that corrects flow nonuniformities can be constructed.

Example Calculations

We now illustrate the variations of the choker bar taper function $l_3(y)$ and $w_2(y)$ by some sample calculations. The following geometric parameters of a linearly tapered coat-hanger die are selected for demonstration:

1. $L = 75$ cm;
2. $w_1 = 0.15$ cm;
3. $w_2 = 0.075$ cm;
4. $l_{30} = 0.5$ cm or 1.0 cm;
5. $\theta = 5°$;
6. $\lambda = \left[3.8214^{1/n}\left(\frac{1.5834}{n} + 5.9005\right)\right]^{-1}$, for a 60° teardrop-shaped manifold[7]; and
7. $h_0 = 1.8283$ cm, for a fluid with the power-law index $n = 0.5$.

Values of the kinematic factor β were reported by Lee and Liu.[10] The coat-hanger die with the above geometric parameters can theoretically deliver a liquid film with perfect flow uniformity. The first-order differential equations that describe the variations of $l_3(y)$ and $w_2(y)$ can be solved by the fourth-order Runge–Kutta method.[28]

How to select a proper choker bar so that the effects of the four production variations on flow nonuniformities can be removed follows.

Enlarged Manifold

Liu et al.[7] found that an enlarged manifold can deteriorate the lateral flow uniformity, and higher flow rates would appear at the ends of the die. The inertial effect was neglected here and then Re = 0. y_0 was selected to be 0.05 and $l_{30} = 0.5$ cm. The effect of the power-law index n on the taper function $l_3(y)$ is displayed in Figure 3. Note here $y = 0$ refers to the die end and $y = 1$ is the die center. $l_3(y)$ is monotonically decreasing from the die end to the center of the die. As n becomes smaller, or the shear-thinning behavior of the fluid is more significant, $l_3(y)$ decreases more rapidly. However as $n = 0.25$, $l_3(y)$ will move up and is close to the curve of $n = 0.75$. It can be observed from Figure 2 that increasing $l_3(y)$ will remove the resistance for fluid to pass through the slot section. The competing effects of n and the geometrical resistance in the slot section will cause $l_3(y)$ to move up as n is small.

The effect of n on the taper function $w_2(y)$ is displayed in Figure 4. As n becomes smaller, $w_2(y)$ will increase monotonically from the die end to the die center. However, as n is smaller than 0.5, the variation of $w_2(y)$ is not obvious as n drops further. The effects of y_0 on $l_3(y)$ and $w_2(y)$ are shown in

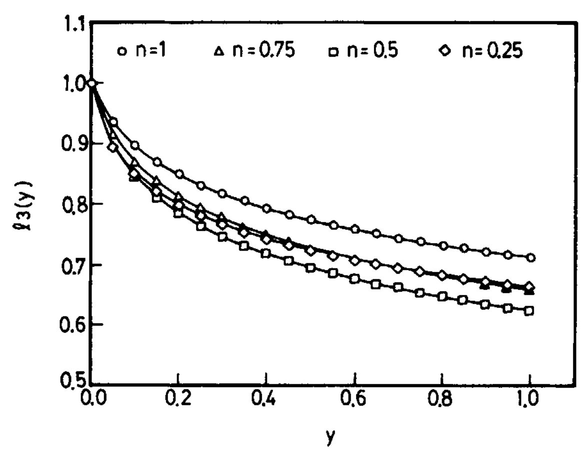

Figure 3 The effect of n on l_3 with $l_{30} = 0.5$ cm, $y_0 = 0.05$, Re = 0, and $w_1 = 2\bar{w}_2$. (○) $n = 1$; (△) $n = 0.75$; (□) $n = 0.5$; (◇) $n = 0.25$.

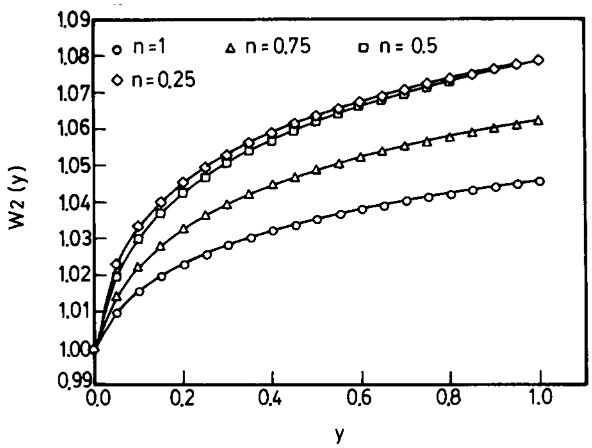

Figure 4 The effect of n on w_2 with $\bar{l}_3 = 1$ cm, $y_0 = 0.05$, Re = 0, and $w_1 = 2w_{20}$. (○) $n = 1$; (△) $n = 0.75$; (□) $n = 0.5$; (◇) $n = 0.25$.

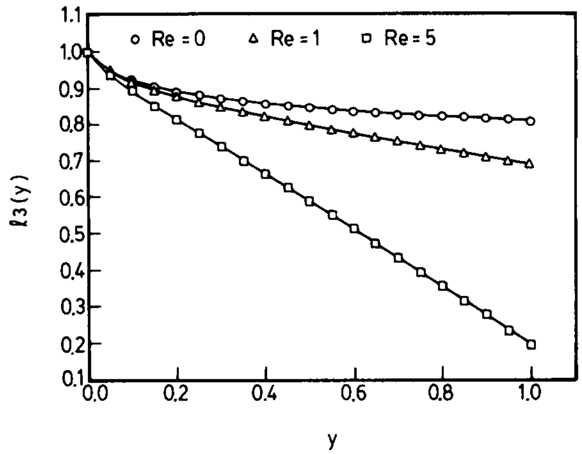

Figure 6 The effect of Re on l_3 with $l_{30} = 1$ cm, $y_0 = 0.05$, $n = 0.5$, and $w_1 = 2\bar{w}_2$. (○) Re = 0; (△) Re = 1; (□) Re = 5.

Figure 5. If y_0 is increased, the variation of $l_3(y)$ and $w_2(y)$ will be more significant.

Inertial Effect

Lee and Liu[10] found that if the fluid inertia becomes significant in a coat-hanger die, the flow rates will increase at two ends of the die and deteriorate flow uniformity. The flow nonuniformities caused by increasing fluid inertia can be corrected by an adjustable choker bar. The variation of $l_3(y)$ as a function of Re is displayed in Figure 6 (note here $y_0 = 0.05$, $l_{30} = 1$ cm and $n = 0.5$). As Re goes up, $l_3(y)$ will decrease monotonically from the die end to the die center. However, as Re is equal to 5, the variation of $l_3(y)$ is quite significant and $l_3(y)$ in the center is only 0.2. The one-dimensional lubrication approach may not be valid if Re is too high; therefore the present design to correct the flow nonuniformities caused by fluid inertia is limited to cases with very low Reynolds numbers. The effect of Re on $w_2(y)$ is displayed in Figure 7. We observe similarly that as Re is greater than 5, the correct approach may no longer be valid.

Variation of Rheological Properties

Liu et al.[7] found that the flow uniformity is very sensitive to the variation of the power-law index n in a linearly tapered coat-hanger die. If a die was originally designed based on $n = 0.5$ and suddenly n was changed to 0.6, a "W" type flow distribution would appear. On the other hand if n dropped to 0.4, an "M" type flow pattern would appear instead.

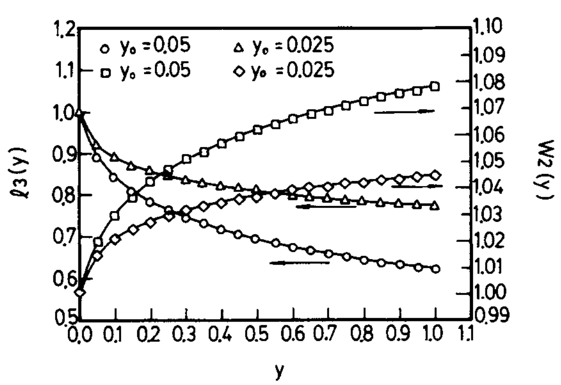

Figure 5 The effect of y_0 on l_3 and w_2 with $y_0 = 0.05$, $n = 0.5$. For l_3: $l_{30} = 0.5$ cm, $w_1 = 2\bar{w}_2$; for w_2: $\bar{l}_3 = 1$ cm, $w_1 = 2w_{20}$. (○) $y_0 = 0.05$; (△) $y_0 = 0.025$; (□) $y_0 = 0.05$; (◇) $y_0 = 0.025$.

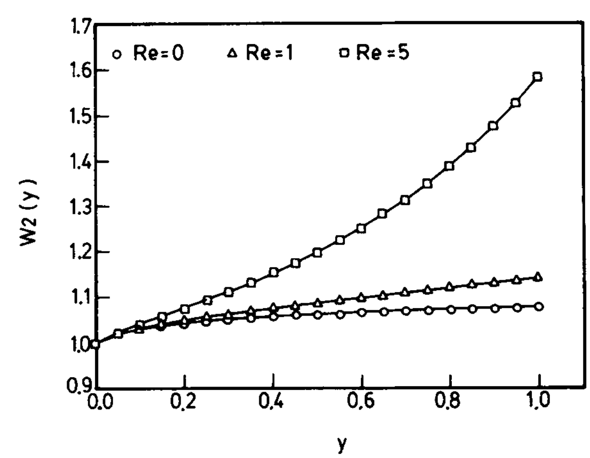

Figure 7 The effect of Re on w_2 with $\bar{l}_3 = 1$ cm, $y_0 = 0.05$, $n = 0.5$, and $w_1 = 2w_{20}$. (○) Re = 0; (△) Re = 1; (□) Re = 5.

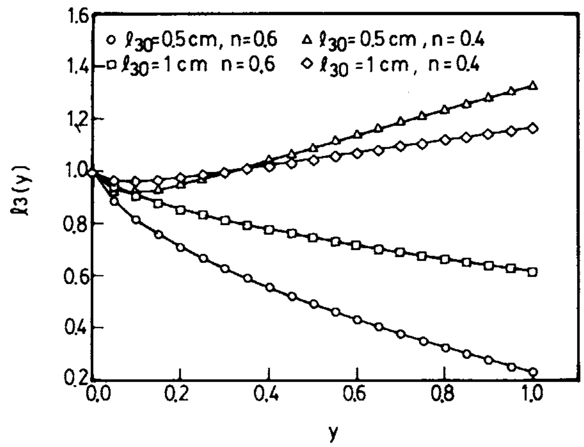

Figure 8 The effect of fluid viscosities on l_3 with $y_0 = 0.05$, $m = 0.5$, Re = 0, and $w_1 = 2\bar{w}_2$. (○) $l_{30} = 0.5$ cm, $n = 0.6$; (△) $l_{30} = 0.5$ cm, $n = 0.4$; (□) $l_{30} = 1$ cm, $n = 0.6$; (◇) $l_{30} = 1$ cm, $n = 0.4$.

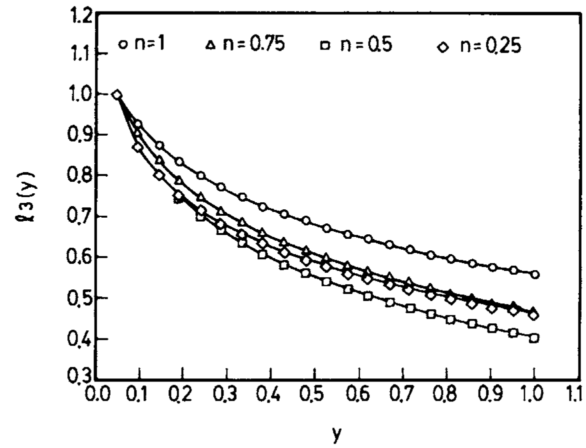

Figure 10 The effect of n on l_3 with $y_0 = 0.05$, $\epsilon = 0.05$, $l_{30} = 0.5$ cm, Re = 0, and $w_1 = 2\bar{w}_2$. (○) $n = 1$; (△) $n = 0.75$; (□) $n = 0.5$; (◇) $n = 0.25$.

The design equations (22) and (24) can be applied to offset the flow uniformities caused by the variation of n. We fixed $n = 0.5$ and made proper selections of h_0 and h. Then n was varied to $n = 0.6$ and 0.4. $l_3(y)$ was computed based on eq. (22) and then a choker bar with the taper function $l_3(y)$ could be constructed to correct flow nonuniformities. The effect of n on $l_3(y)$ is displayed in Figure 8. For the case of $n = 0.6 > 0.5$, $l_3(y)$ has to decrease from the die end to the die center to eliminate the flow nonuniformities. If $n = 0.4 < 0.5$, then $l_3(y)$ will increase from the die end to the die center instead. If l_{30} is longer, the variation of $l_3(y)$ will be smaller. The effect of n on $w_2(y)$ is displayed in Figure 9, and the trend is similar to the case of varying $l_3(y)$.

Production Width Adjustment

If the product has to be narrowed as required, usually the two ends of the die have to be blocked to reduce production width. However, Liu et al.[7] found by doing so, the flow uniformity will deteriorate and higher flow rates will appear at the two ends of the die. To correct the flow nonuniformities caused by narrowing the production width, $l_3(y)$ and $w_2(y)$ can be selected by solving eqs. (27) and (28), respectively. We selected Re = 0, $y_0 = 0.05$, $l_{30} = 0.5$ cm, and the dimensionless reduction of the production width was $\epsilon = 0.05$. The effect of n on $l_3(y)$ is shown in Figure 10; generally $l_3(y)$ is a monotonically decreasing function, as n becomes smaller, $l_3(y)$ will decrease faster. However, as $n = 0.25$, $l_3(y)$ will move up instead. This is similar to the case of en-

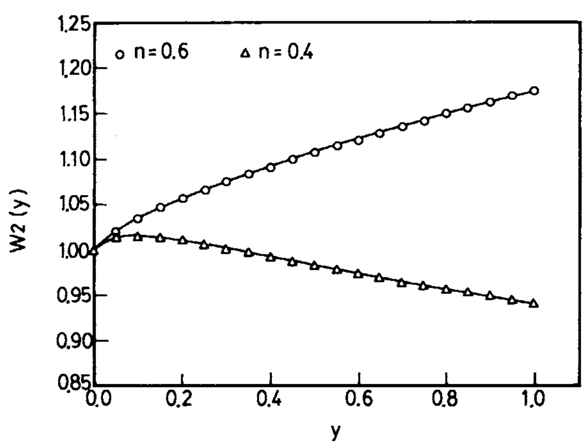

Figure 9 The effect of fluid viscosities on w_2 with $y_0 = 0.05$, $m = 0.5$, Re = 0, $\bar{l}_3 = 1$ cm, and $w_1 = 2w_{20}$. (○) $n = 1$; (△) $n = 0.75$.

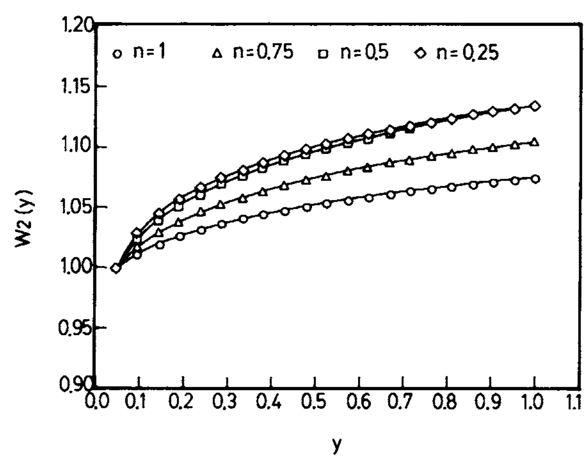

Figure 11 The effect of n on w_2 with $y_0 = 0.05$, $\epsilon = 0.05$, $\bar{l}_3 = 1$ cm, Re = 0, and $w_1 = 2w_{20}$. (○) $n = 1$; (△) $n = 0.75$; (□) $n = 0.5$; (◇) $n = 0.25$.

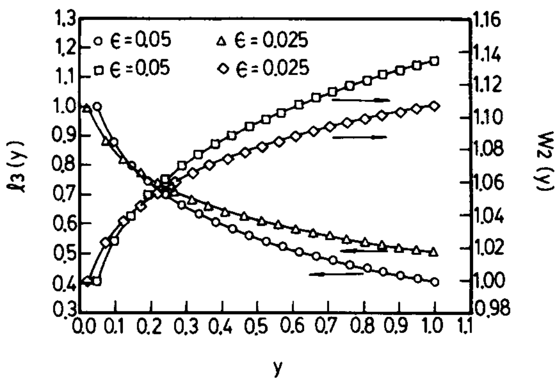

Figure 12 The effect of ϵ on l_3 and w_2 for $y_0 = 0.05$, $n = 0.5$. For l_3: $l_{30} = 0.5$ cm, $w_1 = 2\bar{w}_2$; for w_2: $\bar{l}_3 = 1$ cm, $w_1 = 2w_{20}$. (○) $\epsilon = 0.05$; (△) $\epsilon = 0.025$; (□) $\epsilon = 0.05$; (◇) $\epsilon = 0.025$.

larging the manifold. The effect of n on $w_2(y)$ is displayed in Figure 11, $w_2(y)$ is monotonically increased from the die end to the die center. But as n is smaller than 0.5, the variation of $w_2(y)$ is not obvious. This is also similar to the case of enlarging the manifold. If ϵ is reduced, the variation of $l_3(y)$ and $w_2(y)$ will be decreased accordingly (Fig. 12).

CONCLUSIONS

We described a novel design approach to correct flow nonuniformities caused by four types of production variations in a linearly tapered coat-hanger die. This theoretical approach is based on the one-dimensional lubrication approximation and can be used to predict the taper functions of an adjustable choker bar. Once a choker bar is constructed based on the prediction of the mathematical model, the flow nonuniformities can be properly eliminated by inserting this bar into the die. A choker bar can be tapered in two different ways (Fig. 2). The first method may be easier to machine than the second one. The four production variations we considered include

1. enlarging the manifold;
2. including the fluid inertial terms;
3. varying the viscosity of the polymeric liquids; and
4. narrowing the liquid film width to meet production requirements.

All four production variations can be properly handled, but if the fluid inertia becomes dominant, or Reynolds number is not small, the present method may not be applicable.

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NOMENCLATURE

A	cross-sectional area of the manifold
D	constant in eq. (23)
E	constant in eq. (4)
F_1	variable defined in eq. (6)
F_2	variable defined in eq. (8)
F_3, F_4	variable defined in eq. (15)
f_2, f_3	constants in eq. (9)
g	tan θ
$\bar{h}$, h	square root of A, dimensional and dimensionless
L	die width
l_2, $\bar{l}_3$, l_4	length of land, shown in Figure 1
k	material constant of the power-law model
m, n	power-law index
Nm, Nn, Nv	dimensionless parameters defined in eq. (15)
p	pressure
$\bar{Q}$, Q	flow rate, dimensional and dimensionless
Q_0	inlet flow rate
$\bar{q}$, q	flow rate per unit die width, dimensional and dimensionless
Re	Reynolds number, $\mathrm{Re} \equiv [\rho(Q_0/Lw_1)^{2-n}w_1^n]/k$
v_x	velocity component in the x direction
w_1, $\bar{w}_2$, w_3	slot gaps, shown in Figure 1
(X, Y, Z)	coordinate system, dimensional and dimensionless
(x, y, z)	
y_0	constant in eq. (18)

Greek Letters

β	dimensionless inertial shape factor $\beta = \frac{1}{A}\int \frac{v_x^2}{(\bar{Q}/A)^2}\,dA$
γ	shear rate
δ	constant in eq. (23)
ϵ	dimensionless reduction of the die width
η	viscosity
θ	die angle
λ	dimensionless flow rate
ρ	density

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